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THE EFFECT OF TENSION ON THE TRANSVERSE AND LONGITUDINAL RESISTANCE OF METALS.

BY P. W. BRIDGMAN.

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Introduction.

Although the effect of tension on the resistance of a metal when the current flows in the direction of the tension has been measured by a number of observers,[1] apparently the resistance to current flow at right angles to the direction of the tension has been measured only by Tomlinson,[2] and for only two metals. The matter is one of considerable interest for theories of metallic conduction, and deserves further attention. In the following I give the results of measurements of the transverse coefficient for several metals. I have also measured the longitudinal tension coefficient of the same samples; this is desirable because the longitudinal tension coefficient varies somewhat from sample to sample. In addition to the two tension coefficients, we have available the hydrostatic pressure coefficient[3] (not determined on the identical samples, but not nearly so variable with the specimen) so that the experimental basis for a discussion of the effect of changes of dimensions on resistance is now fairly well laid (at least

for a few metals). It would be desirable if the effect of a pure shear could also be determined, but this is a matter of much greater experimental difficulty.

Method.

The experimental determination of the transverse coefficient is a matter of considerable difficulty. The method employed by Tomlinson[2] apparently leaves much to be desired. His specimen was in the form of a long narrow strip clamped by the long edges so that a tension could be applied crosswise of the strip. The current passed lengthwise of the strip, being led in and out by screw clamps, and the resistance was measured with a Wheatstone's bridge. The state of stress and strain in such a strip is evidently far from simple, because the lateral contraction is hindered, and there are complicated effects under the clamps, which may well constitute a considerable fraction of the whole. It would seem to be a desideratum of a good method that no part of the current which takes part in the measurements should be allowed to flow under clamps.

For the two metals (iron and zinc) which Tomlinson measured, he found that the transverse resistance decreases with tension, the opposite of the longitudinal effect.

In attempting to avoid the difficulties of Tomlinson's method, I adopted a method which demands two different sorts of measurement. The first is the geometrical mean of the longitudinal and transverse tension coefficients, while the second is the longitudinal coefficient alone. From these two data the transverse coefficient may be calculated.

Method of Measuring the Geometrical Mean of the Two Coefficients. In measuring the geometrical mean of the two coefficients, the metal in the form of a thin sheet (from 0.0075 to 0.015 cm. thick) was cut to a rectangle 5 by 10 cm., the two ends gripped by screw clamps, and tension applied by a simple lever arrangement lengthwise of the strip. By way of precaution the strip was insulated from the clamps by strips of mica, although special measurements showed that this precaution was not necessary. The electrical measurements were made on the central portion of the strip. Two sets of measurements were made, one transverse and the other longitudinal. To make either of these sets, a group of 4 contact points was employed, pressed against the strip by springs. These are shown in Figure 1. In making the transverse measurements current was led into the

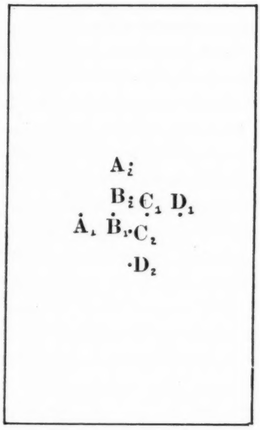

FIGURE 1. Shows the plate with the current and potential terminals. Either the horizontal or the vertical set was used. The current terminals are A and D and the potential terminals B and C. Tension is applied lengthwise of the plate.

plate at A_1 and out at D_1. The difference of potential between the two intermediate contact points B_1 and C_1 was determined on a potentiometer; in this way the specific resistance of the metal per unit of surface may be determined. An exactly similar set of measurements was made with the four vertically disposed contact points. Measurements made with and without tension give the effect of tension.

Without analysis, one might be inclined to think that the measurements with the transversely arranged points would give the transverse coefficient, and those with the longitudinal points the longitudinal coefficient, but detailed mathematical discussion, which will be given later, shows that this is not the case. Except for various corrections, both measurements should give exactly the same thing, namely the geometrical mean of the conductivity per unit surface longitudinally and transversely. At best, then, these two sets of measurements give merely a check on each other, which is not without value. As a matter of fact, however, the various corrections turn out to be so much larger with the longitudinal arrangement, that in making the

final computations only those results were employed which were obtained with the transverse arrangement.

Some of the details of construction require comment. The contact points were large needles pressed against the metal strip with helical springs. The strip was backed with a piece of hard rubber to prevent its bending under the pressure of the contacts. The contact points, with the springs and the backing piece of rubber were constructed as one self-contained unit, which was independently supported, so as to float freely at the center of the metal sheet and follow, without introducing any stresses, the slight displacements due to applying and removing tension. A special holder was used in assembling the apparatus to avoid bending the thin metal sheet. The current and potential leads were flexible copper wire, about 0.015 cm. in diameter, soldered to the needles close to the points so as to minimize thermal e.m.f.'s. Because of the smallness of the effect it was necessary to use rather large currents, of the order of half an ampere. Since the resulting heating effects might be very serious, it was necessary to put the metal sheet with the contact points in a kerosene bath rapidly stirred. Measurements were attempted only at room temperature.

The spacing of the four points was maintained always the same by the use of a special jig; with this, holes were made in a piece of thin cardboard, by which the points were located, and then the cardboard was removed by tearing it apart along cuts previously made in it with a razor blade. Some such use of a jig was necessary, because the needles themselves had to be given some freedom of motion to follow the displacements due to tension. The points were equispaced at approximately 6 mm., making the distance between A_1 and D_1 1.8 cm., and similarly for the longitudinal points.

In terms of the potentiometer measurements the absolute value of the mean surface resistance may be found with and without tension. However, the chief interest of this work is not in values of absolute resistance, but in the percentage changes produced by tension, so that a somewhat simplified and more sensitive procedure in making most of the electrical measurements was adopted. The potential terminals B_1 and C_1 were connected to a senstive moving coil galvanometer (5 cm. deflection per micro-volt). Another e.m.f. was introduced into the same circuit and could be varied by a simple arrangement of variable shunts so that the e.m.f. due to B_1C_1 could be neutralized. The galvanometer zero was noted. Tension is now applied to the specimen; the resistance changes, the e.m.f. due to B_1C_1 changes, and the galvanometer balance is upset, giving a displacement, which

is noted. The load is removed, and again the galvanometer zero noted. This is repeated a number of times, and the mean of the displacements taken. The change of resistance of the specimen corresponding to the galvanometer deflection was determined by finding the deflection caused by known changes of resistance in the balancing circuit. These resistances were so chosen that simple proportionality holds between the deflections and the changes of resistance.

This method would not be applicable unless equilibrium were reached very rapidly. Due to the thinness of the metal sheet and the rapid stirring, all temperature effects arising from the application of tension are dissipated in a time less than that required for the galvanometer to reach equilibrium, so that it is possible to make readings every half minute or less.

It will be seen that the method assumes that the current between the current leads A_1 and D_1 is unaffected by the tension. This was assured by the relative values of the various resistances. The current leads were fed by a storage battery of 12 volts through such a resistance as to cut the current to an ampere or less. The actual resistance between A_1 and D_1 is of the order of a few ten thousandths of an ohm, and the changes in resistance due to tension are of the order of a small fraction of a per cent of this, so that any changes in the feeding current may be entirely neglected.

In addition to these relative measurements of the effect of tension, absolute measurements with the potentiometer were made of the resistance under no load, from which the specific resistance of the metal was computed, and used as a check by comparing with the accepted values for the metal.

In applying tension to the specimen, considerable care is necessary to avoid any shock which might result in permanent set. This was accomplished by attaching the scale pan to the lever by a long helical spring. An arrangement of levers permitted the application and removal of the load by the observer at the galvanometer without changing his position.

Mathematical Theory of the Measurements of the Geometrical Mean Coefficient. The mathematical problem is one of flow in two dimensions in an anisotropic homogeneous substance, the anisotropy consisting of different conductivities in two perpendicular directions. Taking the x and y axes as the axes of principal conductivity (lengthwise and crosswise of our sheet under tension), we have

$$u_x = k_1^2 \frac{\partial \varphi}{\partial x}, \qquad u_y = k_2^2 \frac{\partial \varphi}{\partial y},$$

where u_x and u_y are the components of current, φ the potential, and k_1^2 and k_2^2 the principal conductivities. Since the condition is a steady one we also have the equation of continuity

$$\text{Div } u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0,$$

which, on substituting from above, becomes

$$k_1^2 \frac{\partial^2 \varphi}{\partial x^2} + k_2^2 \frac{\partial^2 \varphi}{\partial y^2} = 0.$$

This may be reduced to the equation for an isotropic medium, the solution of which is known, by the change of variable

$$x = k_1 \xi, \qquad y = k_2 \eta$$

giving

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial^2 \varphi}{\partial \eta^2} = 0.$$

The case in which we are interested is that of a source and a sink symmetrically situated on one of the median lines of a rectangle. We simplify our discussion at first by replacing the rectangle by an infinite plane. The solution may obviously be built up from two point singularities of the type

$$\varphi = -\log r,$$

where

$$r = \sqrt{(\xi - \xi_0)^2 + (\eta - \eta_0)^2}.$$

Consider the auxiliary ξ, η plane, with one point singularity at the origin ($\xi_0 = \eta_0 = 0$), the specific conductivity of the ξ, η plane being unity. Then the solution $\varphi = -\log r$ corresponds to a source at the origin of the auxiliary plane, current flowing uniformly from the source in radial lines to infinity, the total current flowing from the source being 2π. Changing back the variables now gives us a possible solution in the x, y plane, namely

$$\varphi = -\log \sqrt{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}}.$$

We have to ask what is the corresponding current (I) flowing from

the source in the x, y plane. The result may be obtained by integrating u_n over unit circle about the origin.

$$u_n = u_x \cos xn + u_y \cos yn$$

$$= u_x \frac{x}{r} + u_y \frac{y}{r}.$$

Now differentiating φ and substituting above,

$$u_x = \frac{x}{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}}, \qquad u_y = \frac{y}{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}},$$

$$u_n = \frac{r}{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}}.$$

We have to form $I = \int u_n ds$, where ds is the element of arc of unit circle $(x^2 + y^2 = 1)$

$$ds^2 = dx^2 + dy^2, \qquad dy = -\frac{x}{y}\,dx,$$

$$ds = \frac{r}{y}\,dx,$$

$$I = 4\int_0^1 u_n \frac{r}{y}\,dx = 4\int_0^1 \frac{dx}{\sqrt{1-x^2}\left[\frac{1}{k_2^2} + x^2\left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right)\right]}.$$

This integral may be readily evaluated (See e.g. B. O. Pierce's Tables of Integrals, No. 158), giving

$$I = 2\pi k_1 k_2. \tag{1}$$

Now apply this to the special problem in hand, where we have a source and a sink of equal strength. The solution above applies, taking successively the origin at the source and sink, and using the minus sign for the sink. Our problem is to find, for an infinite sheet, the difference of potential between B_1 and C_1 due to a source at A_1

and a sink at D_1 (see figure 1). The distance A_1D_1 is denoted by 2β, and B_1C_1 by 2α. We have at once

Potential at B due to source at A is $-\log\frac{\beta-\alpha}{k_1}$.

Potential at B due to sink at D is $+\log\frac{\beta+\alpha}{k_1}$.

Total potential at B is $\log\frac{\beta+\alpha}{\beta-\alpha}$.

Potential at C due to source at A is $-\log\frac{\beta+\alpha}{k_1}$.

Potential at C due to sink at D is $+\log\frac{\beta-\alpha}{k_1}$.

Total potential at C is $\log\frac{\beta-\alpha}{\beta+\alpha}$.

Whence

$$\varphi_B-\varphi_C=2\log\frac{\beta+\alpha}{\beta-\alpha}. \tag{2}$$

Now the effective resistance between B and C is defined (and measured) as $(\varphi_B-\varphi_C)/I$, which becomes

$$\frac{1}{\pi k_1k_2}\log\frac{\beta+\alpha}{\beta-\alpha} \tag{3}$$

on substituting the values.

The essential feature of this solution is that k_1 and k_2 enter symmetrically, so that by this method only the geometrical mean of the conductivities in two mutually perpendicular directions can be determined.

In applying this mathematical analysis directly to the experimental arrangement two corrections are involved. In the first place, the actual three dimensional flow in the immediate neighborhood of the contact points by which current enters and leaves is replaced in the analysis by a current everywhere two dimensional. A precise mathematical discussion of the correction for this effect is complicated, and in view of the experimental irregularities there is probably little

point in attempting it. The correction is probably beyond the limits of error, because of the thinness of the sheet compared with the distance between the current leads; this ratio was of the order of 1/150 for the metals used. The sign of the effect is to make the specific resistance calculated without correction too large. The effect of tension is to leave this (regarded as a per centage correction) unchanged for the transverse position of the points, but to decrease it slightly for the longitudinal position. The reason for this is that the ratio of the separation of the points to the thickness of the plate is unchanged by tension in the transverse position, but is changed in the longitudinal position.

The second correction is for the finite size of the plate, and is much more important. The rigorous solution for the finite plate may be obtained from the solution above for the infinite sheet by a simple application of the method of images. I owe this idea to a remark of Mr. B. O. Koopman. In applying the method, the plane is cut up into a set of rectangles similar to the original rectangle, with a source or sink in each rectangle in the same position as in the original rectangle. In the strip of rectangles laid off in the direction of the four points the signs alternate, the source being alternately to the right and the left of the sink, whereas in the strip running perpendicularly there is no alternation.

I have carried through an approximate evaluation of the correction for the transverse position of the points. The calculations are not too long with the actual dimensions of the metal sheet used in the experiment. The correction turns out to be approximately 12%, and in this particular case is contributed almost entirely by the phantom rectangles in the strip parallel to the direction of the points, the contributions of all the others nearly cancelling each other and being intrinsically smaller. As a matter of experiment, the mean of the specific resistance of all the metals measured, applying the formula for the infinite plane to the data obtained from the transverse position of the points, was 15% higher than the accepted values, sufficiently good agreement.

The calculation of the correction for the longitudinal position of the points is more difficult, because of slower convergence of the series, and I did not attempt it. It is evident from inspection, however, that the correction will be considerably larger than for the transverse position, and this agrees with the experimental fact that the uncorrected resistance from the longitudinal measurements was 25% high.

Besides determining the correction for the finite size of the sheet, we must also determine how the correction changes when tension is applied, thereby changing the dimensions of the plate, and also making the resistance anisotropic. Here the condition for the transverse position is very much simpler than for the longitudinal position. We have seen that the correction terms are contributed in the transverse position almost entirely by the phantom rectangles extending in the strip parallel to the four points. The terms in the potential arising from the images are logarithms of distances with plus and minus signs (sources and sinks), which may be written in terms of ratios of distances. Under tension, all the effective distances in the strip of phantom rectangles are changed uniformly because of the change of dimensions of the sample, and there is superposed another virtual change, also uniform, because of anisotropy. Hence the ratio of the distances (which enters the expression for the potential) are unchanged and hence the percentage correction on the absolute resistance is unchanged, so that the percentage change of resistance under tension may be calculated without correction from the formulas obtained for the infinite plane. For the longitudinal position, however, the contributions by the strips of phantoms running in both directions must be considered. The distortion is different in different directions, so that there is a correction. This is complicated to compute, and I have not attempted it. One may see by inspection, however, that the uncorrected change of resistance calculated from the longitudinal readings will be too large, and this agrees with the experimental facts.

In the actual calculation of the results, I have used only the data obtained with the transverse position of the points, demanding merely that the longitudinal readings should agree with what might be roughly expected.

Formula 3 yields the two dimensional specific resistance of the metal sheet. In calculating the ordinary resistance per cm. cube, the thickness of the sheet enters. The thickness is changed by tension, so that a correction has to be applied for it, the numerical magnitude of which is evidently σ/E, where σ is Poisson's ratio, and E Young's modulus. The correction is of such a sign that the actually measured proportional increase of resistance is greater than the proportional increase of specific resistance.

Method of Measuring the Longitudinal Coefficient. The measurements just described give the geometrical mean of the longitudinal and transverse coefficients. The straight longitudinal coefficient was measured in much the same way, using the same arrangement of

levers, and kerosene bath, and electrical measuring apparatus. The specimen was a strip 1.25 cm. wide and 10 cm. long, cut lengthwise from the specimens the geometrical mean of which had just been measured. This strip was clamped at either end to screw clamps by means of which the tension was applied, and through which the current was led into and out of the specimen. The potential difference between two points 3 cm. apart in the center of the strip was taken off with needle points pressed against the strip. The dimensions of the strip ensure that at the center the lines of flow are straight and uniformly distributed, so that the change of resistance (measured in terms of galvanometer deflection as above) gives directly the longitudinal effect. The numerical magnitudes are much more favorable here than in the first case, so that it was possible to get the pure longitudinal coefficient with much greater accuracy than the geometrical mean.

In reducing the actually measured proportional changes of resistance to proportional changes of specific resistance, corrections must be applied for the increase of length due to tension and for the lateral contraction in both directions at right angles to the length. The magnitude of the correction is evidently $(1 + 2\sigma)/E$, and is in such a direction as to make the increase of specific resistance with tension less than the measured increase.

Experimental Details.

The experimental details were comparatively straightforward, but much care was necessary because the effects are so small. Greater sensitiveness is obtained by using a greater current, but an upper limit is soon reached because of the heating effects, which increase as the square of the current. The currents were chosen as near the upper limit as feasible. Much may be gained by properly directing the action of the stirrer by which the bath is kept at constant temperature. In applying load to the specimens much care must be taken to remain below the elastic limit, as the properties after even moderate overstrain may be greatly altered. This point is to be emphasized; I believe that in early work it has not been sufficiently regarded. The elastic limit of some of the softer metals in the thorougly annealed condition is surprisingly low. I found it impossible to obtain with this apparatus the effect on aluminum, although many attempts were made, because of the exceedingly low elastic limit of the annealed pure metal. The results obtained with gold and copper

are also exceedingly uncertain from this cause. Silver gave less trouble. The harder metals, iron, nickel, palladium, platinum, gave entirely satisfactory readings, large enough not only to allow a determination of the average value of the coefficients, but also to allow an investigation of the linearity of the relation between stress and change of resistance. For theoretical use I believe it is safe to use the transverse coefficients of only the harder metals, but the longitudinal coefficients of all should be trustworthy.

Aluminum. This was of very unusual purity, containing not over 0.03% total impurity, and was obtained from the same source as the specimens for which I have previously measured the compressibility and the effect of tension on thermal conductivity.[4] It was annealed for several hours at 300°. It was so soft and the elastic limit so low that no measurements of the transverse effect with the four point apparatus could be obtained. A number of attempts were made to obtain the transverse effect, but in all cases loads had to be applied beyond the elastic limit to obtain anything measurable, and the attempt was abandoned. The exceeding of the elastic limit was shown not only by permanent change of dimensions, but also by failure of linearity between stress and change of resistance.

I was able, however, to get better measurements than hitherto of the pure longitudinal effect. This turns out to be very sensitive indeed to slight permanent stretch. The coefficient of a specimen slightly stretched beyond the elastic limit was only one third as great as the highest value obtained on a specimen whose elastic limit had not been exceeded. The maximum stress applied to the best specimen was only 130 kg./cm^2. Within the limits of error, which were somewhat large, the effect is proportional to the load. This is not the case, however, for aluminum which has been strained beyond the elastic limit, the effect departing markedly from linearity and becoming less at the higher stresses.

Two different specimens of aluminum which had apparently not been permanently stretched gave values of the pure longitudinal coefficient not very different from each other, and both were larger than the values which I have obtained before, and larger than values by other observers. The new values are doubtless to be preferred.

Gold. This was sheet 0.012 cm. thick, obtained from Baker and Co. and stated to be of the highest possible purity (guaranteed better than 99.9%). It was annealed by Baker, and also one piece was further annealed by me, but with no perceptible change in the coefficient. Measurements of the purely longitudinal effect were made on three

different specimens with fairly concordant results. I cannot find that the pure longitudinal effect has been previously measured for this metal. The value obtained now is interesting because it is so high. The longitudinal effect is linear with tension with an error not more than 3 or 4% over the range of stress applied here, which was not more than 120 kg./cm^2. The transverse effect for gold was very much less than the longitudinal effect, but is so uncertain that I have not tried to assign any probable value to it.

Copper. Ordinary commercial sheet copper, annealed, was used. There were two different specimens, of thickness 0.012 and 0.20 cm. The measurements of the purely longitudinal effect on the two specimens agreed within 4%, and also agreed with a result which I have previously found for copper of unusually high purity, so that evidently the amount of impurity in commercial copper does not introduce any perceptible error here. Two measurements were also made of the geometrical mean of the two effects; these both agreed in giving a measured effect so small as to be zero within the limits of error. The check measurements with the four points in the longitudinal position behaved as they should for one of the samples, but for the other gave an effect of the wrong sign, for some reason which I have not been able to find. This is the only case of such a discrepancy in all this work, and throws considerable suspicion on the results obtained with the transverse position.

Silver. This was obtained from Baker and Co. of the highest purity, in the form of annealed sheets 0.012 cm. thick. Two samples were used. The behavior when stretched beyond the elastic limit is the same as was found for aluminum, namely the purely longitudinal effect becomes small; in this case about 65% of the effect when the limit is not exceeded. Measurements of the purely longitudinal effect on two different pieces gave coefficients agreeing within less than 1%. The effect is linear within 2% over a tension range of 250 kg./cm^2. The geometrical mean effect was measured on one specimen. It is linear with tension up to 300 kg./cm^2. within an experimental error of 10%.

Platinum. This was annealed sheet, 0.012 cm. thick, of the highest purity (99.9% or better) from Baker and Co. Only one specimen was used. The pure longitudinal effect agreed within about 3.5% with the value which I have previously found for platinum rod. It is linear to better than 1% to 400 kg./cm.^{2}, the highest tension used. The geometrical mean is linear within 4% up to 420 kg./cm.^{2}, and was large enough to give very comfortable measurements. The

behavior for the longitudinal position of the points is as would be expected.

Palladium. This was similar in dimensions to platinum, was obtained from the same source, and was stated to be of the same purity (99.9% or better). There seems, however, for some reason to be considerable difficulty in getting pure palladium, so that I tried to get a check on the purity by measuring the temperature coefficient of resistance of a piece of palladium wire drawn by Baker from metal from the same lot. The coefficient was very low, only 0.0030, so that it would seem that the purity is probably not as high as estimated.

The purely longitudinal coefficient agreed within 4% with the value which I have previously found. The relation is linear with the stress to better than 1% up to 400 kg./cm.², the highest tension used. The geometrical mean coefficient was measured for three different loads to a maximum of 440 kg./cm.² The irregularities were greater than usual, the maximum departure from the linear relation being 15%, but there is no reason to think that this is other than an effect of accidental errors, or that the effect is not actually linear.

Iron. I was not able to obtain pure iron in sheets thin enough for the purpose, and had to content myself with a commercial mild steel of low carbon content. The thickness was 0.010 cm. The dimensions of the original sheet were large enough to allow two samples to be cut, parallel and at right angles to the direction of rolling. This was not possible with the other metals.

The purely longitudinal effect of the two specimens agreed to three significant figures. The value is about 20% lower than I have previously found for iron of very high purity, and agrees essentially with the value found by Tomlinson for iron of no especial purity. The difference is evidently to be ascribed to the carbon content. It is interesting to note that in every case examined so far the effect of impurity is to lower the longitudinal tension coefficient of resistance (if there is any effect at all). Impurity also lowers the temperature coefficient and the pressure coefficient.

The geometrical mean effects for the two specimens did not agree, but was less by 25% for the specimen to which tension was applied in the direction of rolling.

Both effects for both specimens were linear with tension. Because of the largeness of the effect it was possible to make a rather careful examination of this point. The greatest departure of any reading, of which there were 12 in all, from linearity was 5%. The maximum load used was 250 kg./cm.²

Nickel. Two specimens were used from two different sources. Both were what is known commercially as pure nickel, and were somewhat over 99% pure nickel. Both were in the form of sheet 0.012 cm. thick, and were annealed.

It is well known that the pure longitudinal effect is abnormal both with respect to sign, and departure from linearity, and hysteresis. The results obtained with any single specimen are a strong function of the past history. The disentanglement of all the complicated effects in nickel would constitute an elaborate study, and was much beyond the scope of this work. I contented myself here with merely determining whether the sign of the transverse effect would also be found to be abnormal.

The results found with the two specimens agreed in character, although the numerical agreement is not close. It is not worth while to try to reproduce here the complicated results found, but I will merely indicate the general nature.

Both purely longitudinal and geometrical mean effects show hysteresis, saturation (or more probably a maximum), and complicated dependence on the past history. The following numerical values were found for one of the samples. Under a load of 378 kg./cm.^{2} the measured purely longitudinal effect was a decrease of resistance of 0.00540. Correcting for distortion, this becomes a proportional decrease of specific resistance of 0.00570 for this load. Under the same load the geometrical mean of longitudinal and transverse effects was found to be -0.00189. Correcting for distortion this becomes -0.00200. This is less than half the pure longitudinal coefficient, so that it is evident that the transverse specific resistance increases under tension. Detailed calculation gives a proportional increase of transverse specific resistance by 0.00170 for the load of 378 kg./cm.^{2}

The chief conclusion to be drawn from these measurements on nickel is that the longitudinal and transverse coefficients are of opposite sign. Because of the abnormal character of nickel, I shall not attempt to discuss further the significance of the results.

Numerical Results.

In Table I are reproduced the numerical results found for all the metals except nickel. In column 1 are given the names of the metals, in column 2 the measured longitudinal effect, expressed as fractional change of resistance for a tension of 1 kg./cm.^{2}, in column 3 the meas-

TABLE I.

COLLECTED NUMERICAL RESULTS.

Metal	Longitudinal Fractional Change of Resistance, for 1 kg./cm.²		Geometrical Mean of Longitudinal and Transverse Fractional Change of Resistance for 1 kg./cm.²		Calculated Fractional Change of Transverse Resistance for 1 kg./cm.²
	Measured	Corrected for Distortion	Measured	Corrected for Distortion	
Al	$+5.8 \times 10^{-6}$ 6.9	$+4.0 \times 10^{-6}$			
Au	$+6.25 \times 10^{-6}$ 5.94 6.33	$+3.87 \times 10^{-6}$	Not greater than $+0.40 \times 10^{-6}$	$-.17 \times 10^{-6}$ (upper limit)	-4.5×10^{-6} (very uncertain)
Cu	$+3.01 \times 10^{-6}$ 3.15	$+1.75 \times 10^{-6}$	$-.1$ to $.0 \times 10^{-6}$	$-.3 \times 10^{-6}$	-2.4×10^{-6}(?)
Ag	$+5.06 \times 10^{-6}$ 5.03	$+2.86 \times 10^{-6}$	$+1.89 \times 10^{-6}$	$+1.41 \times 10^{-6}$	$-.04 \times 10^{-6}$
Pt	$+2.82 \times 10^{-6}$	$+1.78 \times 10^{-6}$	$+1.29 \times 10^{-6}$	$+1.06 \times 10^{-6}$	$+.34 \times 10^{-6}$
Pd	$+2.90 \times 10^{-6}$	$+1.37 \times 10^{-6}$	$+1.29 \times 10^{-6}$	$+.94 \times 10^{-6}$	$+.51 \times 10^{-6}$
Fe (1)	$+2.13 \times 10^{-6}$	$+1.42 \times 10^{-6}$	$+.99 \times 10^{-6}$	$+.86 \times 10^{-6}$	$+.30 \times 10^{-6}$
Fe (2)	$+2.13 \times 10^{-6}$		$+1.33 \times 10^{-6}$	$+1.20 \times 10^{-6}$	$+.78 \times 10^{-6}$

(1) Tension parallel to direction of rolling.
(2) Tension perpendicular to direction of rolling.

ured results of column 2 are corrected for the distortion by tension so as to give the change of specific resistance for a tension of 1 kg./cm.² The correction by which column 3 is obtained from column 2 is $(1 + 2\sigma)/E$. Column 4 contains the measured change of resistance as given by the four point apparatus with the points in the transverse position. In column 5 the results of column 4 are corrected for the distortion produced by tension by subtracting the term σ/E, giving the geometrical mean of the effect of 1 kg./cm.² on longitudinal and transverse resistance. Finally, in column 6 is given the effect of tension on specific resistance transverse to the direction of the tension. Column 6 is obtained by subtracting from column 5 half of column 3 and multiplying by 2. The essentially new results, to obtain which this investigation was made, are those contained in column 6; the results of column 3 are also necessary for any theoretical discussion.

The transverse coefficient may have either sign. The cases of negative sign are much less certain than the positive cases. The negative value for gold is very uncertain, that for silver is so small that within experimental error it might well be positive, and the value for copper is also very uncertain, as may be seen by examining the measured geometrical mean effects. It is perhaps significant that the negative coefficients are shown by the softer metals. The positive values for palladium, platinum and iron, are, however, much more certain experimentally; the effects were larger, larger stresses could be applied, and the corrections for distortions were smaller and more certain. The positive sign is the reverse of that found by Tomlinson in the two cases examined by him.

Theoretical Discussion.

In the first place, we are now in a position to answer a question of somewhat formal character raised in connection with the effect of tension on the resistance of the abnormal metals, namely whether it is possible to connect the changes of resistance directly with the changes of dimensions, changes of dimensions at right angles to the direction of current flow affecting the resistance differently from changes parallel to the flow. The coefficient of proportionality may be written k_l for parallel (longitudinal) changes, and k_t for perpendicular (transverse) changes. This involves in general writing for the change of resistance;

$$\frac{\Delta R}{R} = k_l \frac{\Delta \delta_l}{\delta_l} + k_t \left[\left(\frac{\Delta \delta_t}{\delta_t} \right)_1 + \left(\frac{\Delta \delta_t}{\delta_t} \right)_2 \right],$$

where $\Delta\delta_l/\delta_l$ denotes the strain in the direction of flow, and $\Delta\delta_t/\delta_t$ that at right angles to it, the latter appearing twice because there are two directions perpendicular to the flow. The equation as assumed involves two coefficients. The assumption may be checked by comparing the values of $\Delta R/R$ given by three independent kinds of experiment, which the data of this paper now for the first time place at our disposal, namely the changes of resistance under hydrostatic pressure (given by previous work), longitudinal changes under tension, and transverse changes under tension. For these three kinds of stress we have the following deformation:

Hydrostatic pressure

$$\frac{\Delta\delta_l}{\delta_l} = \frac{\Delta\delta_t}{\delta_t} = \frac{1}{3}\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t p.$$

Tension, longitudinal effect

$$\frac{\Delta\delta_l}{\delta_l} = \frac{T}{E}, \qquad \left(\frac{\Delta\delta_t}{\delta_t}\right)_1 = \left(\frac{\Delta\delta_t}{\delta_t}\right)_2 = -\frac{\sigma T}{E}. \qquad (T \text{ is tension.})$$

Tension, transverse effect

$$\frac{\Delta\delta_l}{\delta_l} = -\frac{\sigma T}{E}, \qquad \left(\frac{\Delta\delta_t}{\delta_t}\right)_1 = \frac{T}{E}, \qquad \left(\frac{\Delta\delta_t}{\delta_t}\right)_2 = -\frac{\sigma T}{E}.$$

Hence we have the following three equations for the two coefficients k_l and k_t.

Hydrostatic pressure

$$\frac{1}{p\dfrac{1}{3}\dfrac{1}{v}\left(\dfrac{\partial v}{\partial p}\right)_t} \cdot \frac{\Delta R}{R} = k_l + 2k_t.$$

Tension, longitudinal

$$\frac{E}{T}\cdot\frac{\Delta R}{R} = k_l - 2\sigma k_t.$$

Tension, transverse

$$\frac{E}{T}\cdot\frac{\Delta R}{R} = -\sigma k_l + k_t(1-\sigma).$$

Putting p and T equal unity gives to $\Delta R/R$ the values of pressure or tension coefficient. The values used in checking the equations are given in Table II.

TABLE II.

Metal	Pressure Coefficient of Specific Resistance	Longitudinal Tension Coefficient of Specific Resistance	Transverse Tension Coefficient of Specific Resistance	Linear Compressibility $-\frac{1}{3}\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t$	$\frac{1}{E}$	σ	k_l	k_t	$-\sigma k_l+(1-\sigma)k_t$ Obs.	Calc.
Au	-3.33×10^{-6}	$+3.87 \times 10^{-6}$	-4.9×10^{-6}	$.193 \times 10^{-6}$	1.25×10^{-6}	.42	7.3	5.0	−3.92	−0.17
Ag	−3.83	+2.86	− .04	.329	1.26	.38	4.84	3.38	− .03	+ .26
Cu	−2.45	+1.75	−2.4	.240	.81	.34	4.20	3.00	−2.96	+ .55
Pd	−2.16	+1.37	+ .51	.173	.89	.39	4.62	3.94	+ .57	+ .60
Pt	−2.07	+1.78	+ .34	.120	.59	.39	7.0	5.1	+ .58	+ .38
Fe	−2.60	+1.42	+ .54	.196	.48	.28	5.2	4.05	+1.13	+1.45
Al	−4.87	+4.0		.448	1.42	.34				

The check is applied by comparing the observed value of $-\sigma k_l + (1 - \sigma)k_t$ with the calculated value. The check is probably as good as could be expected for Pt, Pd, and Fe, the metals for which the transverse coefficient was determined with the greatest accuracy, is not bad for Ag, considering that the calculated value is the difference of two numbers nearly equal, while for Au and Cu there is no agreement, but the experimental values were also exceedingly uncertain. I believe that these results give rather high probability to the legitimateness of the assumption made as to the connection between distortion and change of resistance, and that for the soft metals, for which the transverse effect is exceedingly difficult to measure, the transverse effect may probably be calculated from the hydrostatic pressure and longitudinal tension effects with greater accuracy than it can be measured.

The numerical values of k_l and k_t are of interest. In all cases they are positive (that is, increasing the distance between atoms increases resistance), and k_t is numerically less than k_l. It is perhaps at first unexpected that k_t is not zero. A reason is at once suggested by any free path theory of conduction, for the motion of those electrons which are in paths inclined to the e.m.f. is affected by transverse strain because the transverse strain has a component along the path.

This formal expression for the change of resistance in terms of strain allows considerable latitude in the underlying physical picture. For instance, if the free flight is from atom to atom, through the substance of the atom, as I have supposed in my theory for normal metals, the coefficient k_l would be expected to be the most important and positive, whereas if the path lies between the atoms, k_t would be most important and negative. The values found above correspond to the expectation for a normal metal.

It is one of the tasks of any complete theory to reproduce the experimental connection between these three effects, or to reproduce k_l and k_t. This I believe no theory is at present in a position to attempt, but we may get some indications of the relative magnitudes of the transverse and longitudinal effects. I have already given some discussion of this question in connection with the longitudinal tension coefficient. The conclusion there drawn was that the classical free electron theory, in which the electron is free to drift in the direction of an applied e.m.f. irrespective of the direction of the free path relative to the e.m.f., must be modified by picturing the electron as moving in something like a fixed groove, so that the velocity of drift

imparted by the applied e.m.f. is directed along the original path, and is effectually produced only by the component of e.m.f. along the path.

This conclusion is much strengthened by the new evidence of this paper. There was, however, an error in the mathematical analysis of the former paper[5] which must be corrected. The problem is to integrate the effect of strain over all directions. The physical assumption made is that the change in length of any electronic free path is proportional to the geometrical elongation in that direction. In terms of my previous theory, this means a normal metal, in which there are no free paths between the atoms. It is also assumed that when a body is strained there is no redistribution of the relative number of electronic paths in different directions, nor any redistribution of velocity. This amounts to maintaining one of the fundamental assumptions of the classical theory, namely that after the free paths of the electrons have been terminated by collision, the new distribution of velocities is entirely at random. This means that the entire effect on resistance produced by strain is due to an alteration of length of free path.

Consider now the strain due to tension. In the direction of the tension there is an elongation

$$\frac{\delta l}{l} = \frac{T}{E},$$

and at right angles to the tension a contraction

$$\frac{\delta l}{l} = -\sigma \frac{T}{E}.$$

For any intermediate direction, making an angle θ with the tension, the elongation is

$$\frac{\delta l}{l} = \frac{T}{E}(\cos^2\theta - \sigma\sin^2\theta).$$

In the preceeding paper an incorrect expression, $\frac{T}{E}[1 - (1 + \sigma)\sin\theta]$ was used for this. We now assume that the free path, L, of an electron in any direction in the strained metal is related to the free path in the unstrained metal, L_0, by the equation

$$L = L_0\left(1 + \alpha\frac{\delta l}{l}\right) = L_0\left[1 + \alpha\frac{T}{E}(\cos^2\theta - \sigma\sin^2\theta)\right],$$

where α is an empirical constant, and may most easily be evaluated in terms of the pressure coefficient of resistance.

Consider now a metal to which an e.m.f. $\mathcal{E}$ is applied, and an electron moving in a free path making an angle φ with $\mathcal{E}$. The conventional free electron analysis gives for the velocity of drift in the direction of $\mathcal{E}$ the expression $\frac{1}{2}\frac{\mathcal{E}e}{m}\cdot\frac{L}{v}$, provided the force $\mathcal{E}$ produces full drift in its own direction. Here v is the normal electron velocity, supposed large compared with the velocity of drift. The result is obtained as follows. The time of free flight is L/v, the force acting is $\mathcal{E}e$, the acceleration is $\mathcal{E}e/m$, the velocity of drift at the end of free flight is $\frac{L}{v}\cdot\frac{\mathcal{E}e}{m}$, and hence the average velocity of drift is $\frac{1}{2}\frac{\mathcal{E}e}{m}\frac{L}{v}$. The total current is obtained by integrating, or

$$I = \int_0^{2\pi} d\psi \int_0^{\pi} n\frac{\mathcal{E}e}{m}\cdot\frac{L_0}{v}\sin\theta\left[1 + \alpha\frac{T}{E}(\cos^2\theta - \sigma\sin^2\theta)\right]d\theta,$$

where θ and ψ are ordinary polar coordinates, and n is the number of free electrons per unit volume per unit solid angle. The conductivity is obtained from this equation by dividing both sides by $\mathcal{E}$, or by setting $\mathcal{E}$ equal to unity.

But if the electron is not free to drift completely in the direction $\mathcal{E}$, but is constrained to maintain its original direction, then the component drift in the direction of $\mathcal{E}$ is $\frac{1}{2}\frac{\mathcal{E}e}{m}\cdot\frac{L}{v}\cdot\cos^2\varphi$, $\cos\varphi$ entering once because $\mathcal{E}$ must be resolved along the path, and again because the drift so produced must be resolved along $\mathcal{E}$. The conductivity is now

$$X = \int_0^{2\pi} d\psi \int_0^{\pi} n\frac{eL_0}{mv}\sin\theta\cos^2\varphi\left[1 + \alpha\frac{T}{E}(\cos^2\theta - \sigma\sin^2\theta)\right]d\theta.$$

Our problem is now to evaluate these expressions corresponding to different special cases. First consider the expression given by the hypothesis of unrestrained drift. It is at once evident that the conductivity is independent of the direction of $\mathcal{E}$, that is, the transverse change of resistance is the same as the longitudinal change. The reason of course, is that under the assumptions every electron makes a contribution to the total conductivity determined only by its time

of free flight, and independent of the direction of $\mathcal{E}$. *This conclusion is directly contrary to the experimental evidence,* and demands that we abandon this conception of free flight. The same assumptions would also lead us to the conclusion that in a crystal the resistance must be the same in all directions, which again is directly contradicted by experiment.

For convenience of reference we record the result of carrying out the integration on the hypothesis of unrestricted drift. The result is

$$X_l = X_t \backsim 1 + \frac{1}{3}\,\alpha\,\frac{T}{E}(1-2\sigma), \quad \text{unguided path.}$$

Turn now to the assumption of guided paths. We have to consider two cases, $\mathcal{E}$ along the tension, or at right angles. In the first case $\cos\varphi = \cos\theta$, and in the second $\cos\varphi = \sin\theta\cos\psi$. Hence we have respectively for the longitudinal and transverse conductivities

$$\left.\begin{aligned} X_l &\backsim 1 + \frac{3}{5}\,\alpha\,\frac{T}{E}\left(1 - \frac{2\sigma}{3}\right) \\ X_t &\backsim 1 + \frac{1}{5}\,\alpha\,\frac{T}{E}(1-4\sigma) \end{aligned}\right\}\text{guided path.}$$

For pure hydrostatic pressure, $L = L_0(1 + \alpha kp)$, where k is the linear compressibility. The two hypotheses give

$$Xp \backsim \int_0^{2\pi} d\psi \int_0^{\pi} \sin\theta(1+\alpha kp)d\theta \backsim 1 + \alpha kp, \quad \text{unguided path,}$$

$$Xp \backsim \int_0^{2\pi} d\psi \int_0^{\pi} \sin\theta\cos^2\theta(1+\alpha kp)d\theta \backsim 1 + \alpha kp, \quad \text{guided path.}$$

In applying this analysis, we have assumed n unaffected by the strain. But now it has been one of the assumptions of our theory that the number of free electrons *per atom* is constant. This means that in making experimental comparisons we must use the various coefficients "corrected for change of volume," as was done in the 1923 paper.

We may first apply the formulas given above to correct the previous results.[5] The ratio of pressure coefficient to longitudinal tension coefficient for the unguided path should be

$$\frac{3kE}{1-2\sigma}.$$

The sign of this does not agree with experiment, as already indicated.

On the guided path theory, the ratio is

$$\frac{5kE}{3\left(1-\frac{2\sigma}{3}\right)}.$$

Assuming for σ the mean value 1/3, this is $(15/7)kE$. The value previously found was $4.7kE$.

Using the new data, we now compare this expression with experiment. The results are given in Table III, columns 4 and 5. The agreement is not good. It is to be noticed, however, that the new value for the longitudinal tension coefficient of aluminum removes aluminum from a markedly exceptional position. In the last column of the table, is given the corrected transverse coefficient, X_t, for purposes of reference.

Returning now to the expressions for X_l and X_t on the guided path theory, we see that there is provision for a variation of sign. X_l is always positive for those normal metals whose resistance decreases under hydrostatic pressure. The reason is that $1 - 2\sigma/3$ is always positive, the maximum value of σ consistent with stability being 0.5. X_t on the other hand, is negative if σ is greater than 0.25. Now σ is greater than 0.25 for all the metals listed here, but nevertheless X_t is positive for the more certain ones. It is evident, therefore, that the picture of the free path given above does not correspond exactly to the facts, but it must be credited with making a step in the right direction in allowing either sign.

It is evident that a closer approach to experiment may be made by taking some sort of a mean between the guided and the unguided path formulas. The physical significance of this may be perhaps that both effects are present.

We turn now to another question connected with the tension coefficient, namely connection with crystal symmetry. The metals examined in this paper all belong to the cubic system crystallographically, and may therefore be expected to show a certain simplicity as compared with other metals. It should nevertheless be recognized that even with the cubic metals crystal structure has an effect, and that the quantities X_l and X_t may be expected to be different for samples cut in different directions from a single crystal. The reason of course is that the strain produced by tension is a function of the orientation. The values found above experimentally are mean

TABLE III.

Comparison of Experimental and Calculated Relation between Pressure and Longitudinal Tension Coefficients of Resistance.

Metal	Pressure Coefficient of Specific Resistance Corrected by $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t$	Longitudinal Tension Coefficient of Specific Resistance corrected by $\frac{1-2\sigma}{E}, X_l$	Ratio, $\frac{\text{Corrected Press. Coeff.}}{\text{Corrected L. Ten. Coeff.}}$	$\frac{5}{3}\frac{kE}{1-\frac{2\sigma}{3}}$	Transverse Tension Coefficient of Specific Resistance, corrected by $\frac{1-2\sigma}{E}, X_t$
Al	-3.97×10^{-6}	$+3.55 \times 10^{-6}$	1.12	.68	——
Au	-2.75	$+3.65$	.75	.37	——
Ag	-2.83	$+2.56$	1.10	.58	$-.34 \times 10^{-6}$
Cu	-1.73	$+2.19$	.79	.64	$-.27$
Pd	-1.64	$+1.96$	.84	.44	$+.31$
Pt	-1.71	$+1.94$	.88	.46	$+.21$
Fe	-2.01	$+2.39$	.84	.84	$+.33$

values. The pressure coefficient of resistance, on the other hand, is independent of the orientation of the specimen. Experimentally we may have a manifestation of the effect in the different results obtained for iron parallel and perpendicular to the direction of rolling.

The general problem of determining the number of constants required to completely determine the resistance in all directions for any system of stress in a single crystal belonging to any crystallographic system does not seem to have been discussed.

Summary

A new experimental method for measuring the change of resistance in a metal under tension when the direction of flow is at right angles to the tension (transverse coefficient) has been developed. The method gives immediately the geometrical mean of the transverse and longitudinal coefficients. An independent determination of the longitudinal coefficient permits a calculation of the transverse coefficient. Measurements are presented for 8 metals. Nickel is abnormal, as it also is with respect to the longitudinal coefficient. It is established that the signs of the two coefficients are opposite for Ni. The values found for the softer of the other metals, Al, Au, Ag, and Cu, are uncertain because of the smallness of the effect and the necessity for remaining below the elastic limit. It is probable that the transverse coefficients of some of these metals are negative. The results for Pd, Pt, and Fe are much more certain. The transverse coefficients of these are positive; this sign is the opposite of that found by the only previous observer, Tomlinson.

In the discussion it is shown that probably all changes of resistance due to deformation at constant temperature may be described in terms of two coefficients, one connecting the change of resistance with changes of dimensions at right angles to the current flow, and the other connecting with changes of dimension parallel to the flow. It is the task of theory to reproduce these two coefficients. It is pointed out that if some form of free path theory of conductivity is maintained it is not possible to suppose the electrons free to drift unrestrainedly in the direction of the applied force, for under such conditions the transverse and longitudinal coefficients are equal. It must be, therefore, that the electrons are constrained to a certain extent to move along guided paths in the metal. No theory is at present able to account satisfactorily for all the deformational effects. With regard to my own particular form of free path theory, a cor-

rection in a previous calculation of the longitudinal tension coefficient from the pressure coefficient is made by which the agreement between calculation and experiment becomes less good. It must be recognized, however, that this form of theory is so far in accord with the facts as it allows a transverse coefficient of either the same or opposite sign from that of the longitudinal coefficient.

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